The geometry of consumer preference aggregation

Fedor Sandomirskiy (Caltech) **Philip Ushchev** (ECARES, U.libre de Bruxelles)

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from individual to population behavior and back

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- **Our paper is a middle ground:** a rich enough tractable setting

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Brings information economic tools to the classical problem

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a method linking individual characteristics and market demand properties

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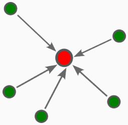

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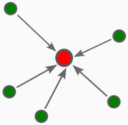
- enables extreme-point and convexification tools



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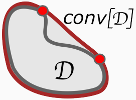

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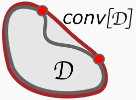

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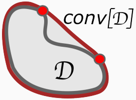

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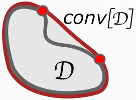

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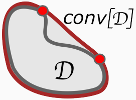

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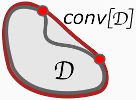

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Identification of preference distributions

- Aggregate behavior pins down preference distributions for "simplex domains"

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- **Economic applications of extreme points, Choquet theory, and convexification**
  - Kleiner et al. (2021), Arieli et al. (2020), Manelli & Vincent (2010), Kamenica & Gentzkow (2011), Aumann et al. (1995)

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- demand as a function of prices $\mathbf{p}$

$$D(\mathbf{p}, b) = \operatorname*{arg\,max}_{\mathbf{x} \in \mathbb{R}^n_+ \,:\, \langle \mathbf{p}, \mathbf{x} \rangle \leq b} u(\mathbf{x})$$

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Challenging problem, no structural insights

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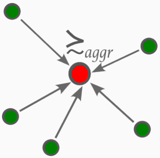

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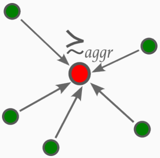

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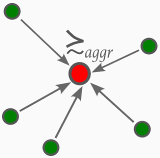

- The dual to Eisenberg-Gale
- A simple result with numerous implications

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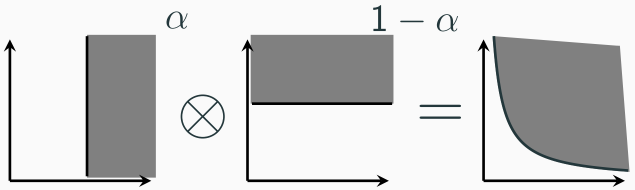
Corollary

The upper contour set of the aggregate consumer is the geometric mean of individual upper contour sets

$$\left\{u_{\mathrm{aggr}}(\mathbf{x}) \geq 1\right\} = \left\{u_1(\mathbf{x}) \geq 1\right\}^{\beta_1} \otimes \left\{u_2(\mathbf{x}) \geq 1\right\}^{\beta_2} \otimes \ldots \otimes \left\{u_m(\mathbf{x}) \geq 1\right\}^{\beta_k}$$

Example: single-minded consumers



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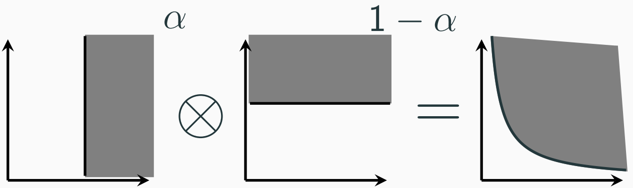

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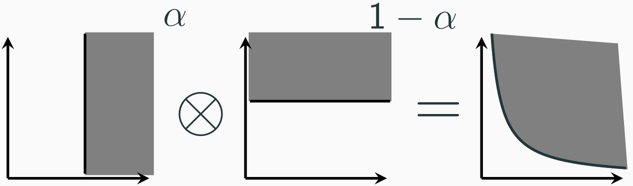

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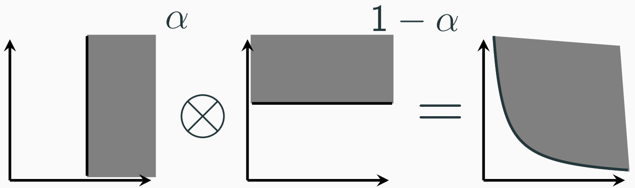

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- **Algebra:** $\alpha \cdot \log p_1 + (1-\alpha) \cdot \log p_2 = \log\left(p_1^{\alpha} \cdot p_2^{1-\alpha}\right)$
- **Economics:** two single-minded consumers generate the same demand as one Cobb-Douglas consumer $u(\mathbf{x}) = x_1^{\alpha} \cdot x_2^{1-\alpha}$

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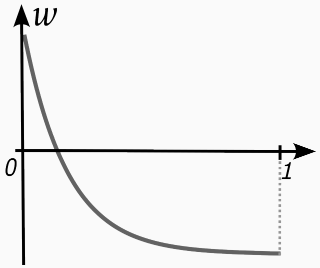

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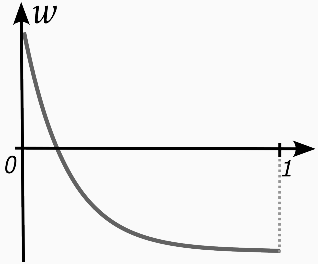

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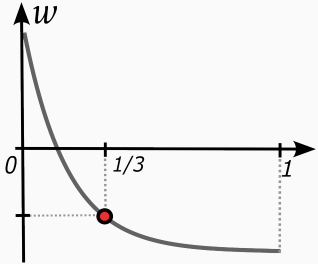

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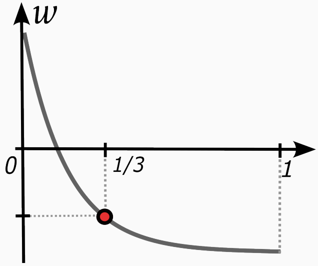

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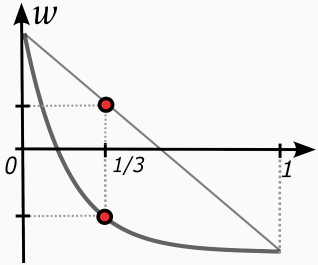

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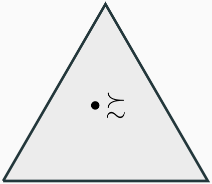

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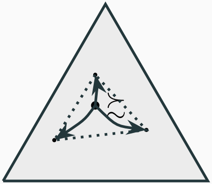

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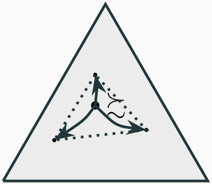

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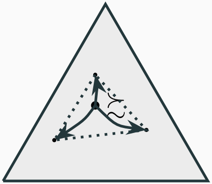

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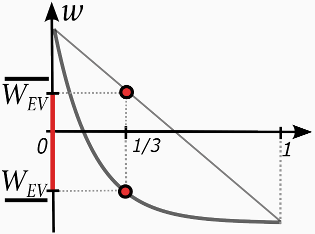

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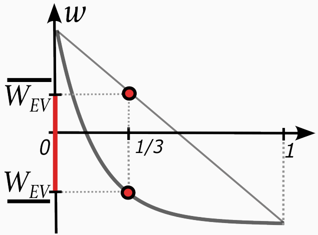

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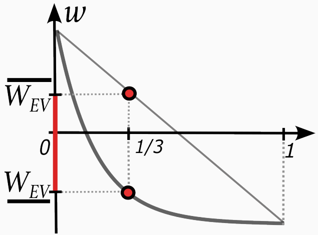

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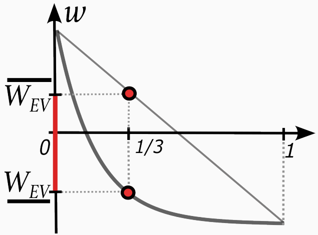

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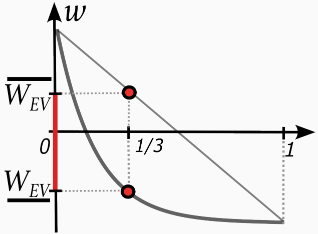

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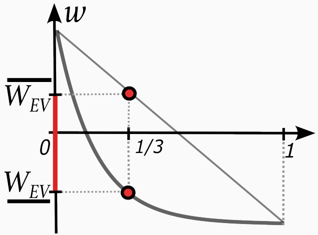

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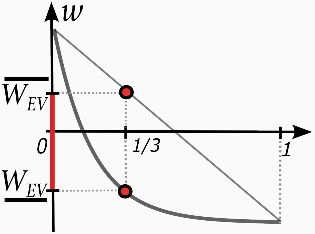

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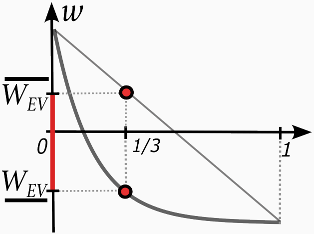

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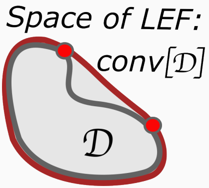

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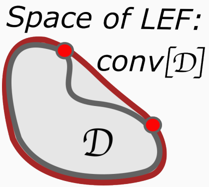

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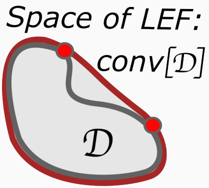

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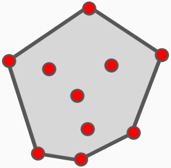

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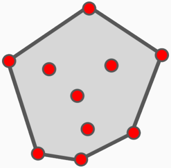

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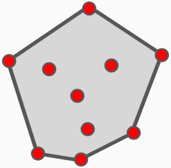
Finitely-generated domains



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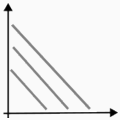

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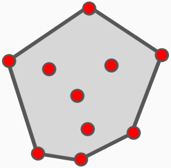

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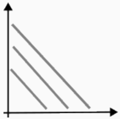

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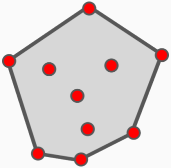

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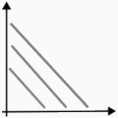

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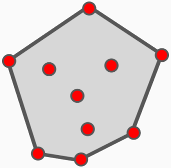

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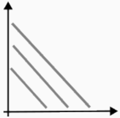

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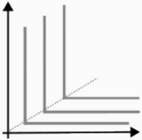

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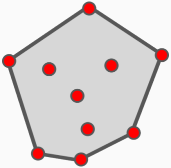

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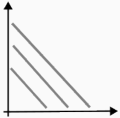

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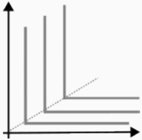

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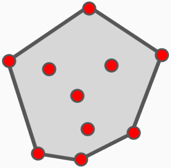

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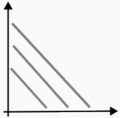

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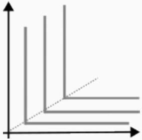

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Conclusion

Parameters are not aligned with aggregation $\Rightarrow$ large completion

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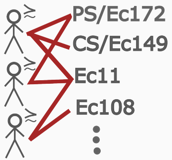

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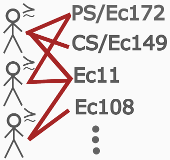

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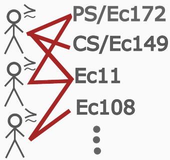

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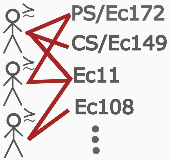

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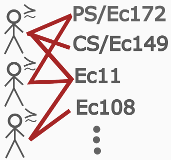

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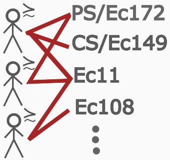

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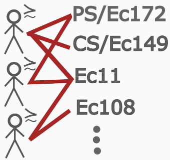

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Conclusion

Use finitely-generated $\mathcal{D}$ as bidding languages in large-scale applications

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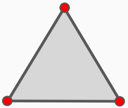

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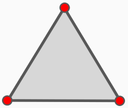

Examples:

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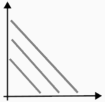
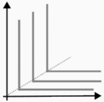


Leontief for $n = 2$ goods



Key takeaways

To handle aggregation, represent preferences by LEF

- All preferences $\simeq$ a compact convex set
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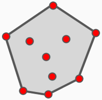

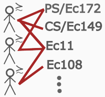

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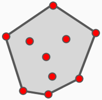

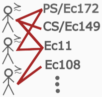

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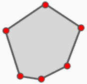

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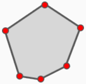

Example: indecomposable preferences over 2 goods

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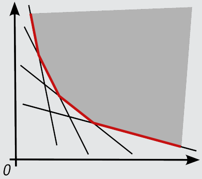

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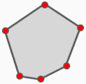

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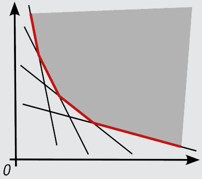

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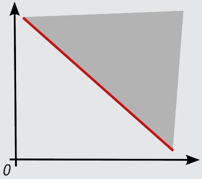

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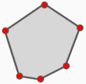

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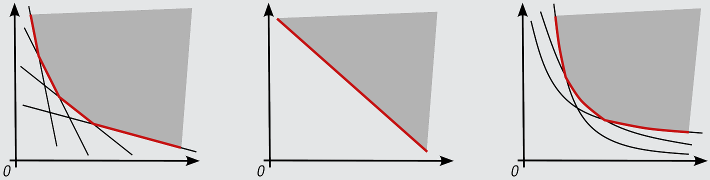

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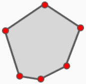

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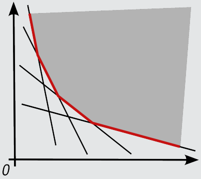

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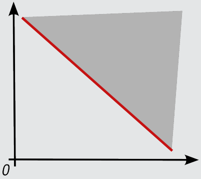

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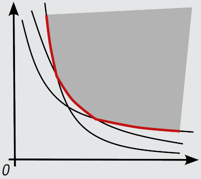

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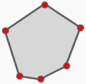

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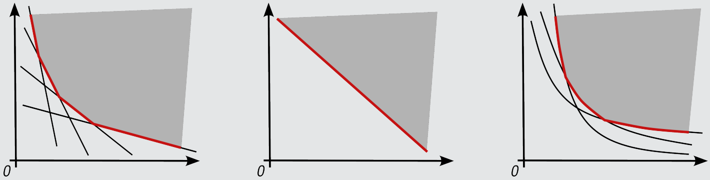

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Conclusion

Indecomposable preferences are "elementary building blocks"

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More Related Literature back

- **Endogenous incomes and general preferences $\Rightarrow$ "anything goes" for aggregate demand:**
  - Sonnenschein (1973), Mantel (1974, 1976), Debreu (1974), Chiappori and Ekeland (1999), Kirman and Koch (1986), Hildenbrand (2014)
- **Representative agent approach**
  - **Criticism of representative agents:** Caselli & Ventura (2000), Carroll (2000), Kirman (1992)
  - **Household behavior:** Samuelson (1956), Chambers and Hayashi (2018), Browning & Chiappori (1998)
- **PIGLOG, AIDS, and similar functional forms**
  - Muellbauer (1975,1976), Deaton & Muellbauer (1980), Lewbel & Pendakur (2009)

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- Closure and the Borel structure are w.r.t. the distance $d(\succsim, \succsim') = \max_{\mathbf{p}\in\Delta_{n-1}} \left| \frac{(\ln E(\mathbf{p}) - \ln E((1,\ldots,1))) - \left(\ln E'(\mathbf{p}) - \ln E'((1,\ldots,1))\right)}{(1+\max_i |\ln p_i|)^2} \right|$
- Preferences form a compact set $\simeq$ convex subset of $C(\Delta_{n-1})$

Integral representation of the completion

For infinite domains, we need to allow "continual" convex combinations

Theorem 3

The completion of $\mathcal{D}$ = preferences with expenditure functions E s.t.

$$\log E(\mathbf{p}) = \int_{\overline{\mathcal{D}}} \log E_{\succsim}(\mathbf{p}) \mathrm{d}\mu(\succsim),$$

where μ is a Borel probability measure supported on the closure $\overline{\mathcal{D}}$ of $\mathcal{D}$

- Closure and the Borel structure are w.r.t. the distance $d(\succsim, \succsim') = \max_{\mathbf{p}\in\Delta_{n-1}} \left| \frac{(\ln E(\mathbf{p}) - \ln E((1,\ldots,1))) - \left(\ln E'(\mathbf{p}) - \ln E'((1,\ldots,1))\right)}{(1+\max_i |\ln p_i|)^2} \right|$
- Preferences form a compact set $\simeq$ convex subset of $C(\Delta_{n-1})$
- Choquet theory $\Rightarrow$ Theorem 3