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PERSUASION AS TRANSPORTATION

N-AGENT PERSUASION

HOW TO SUPPLY INFORMATION OPTIMALLY TO MULTIPLE AGENTS?

today:
two agents, binary state

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state

$$\theta = \begin{cases} h & \text{prob. } p \\ \ell & \text{prob. } 1-p \end{cases}$$

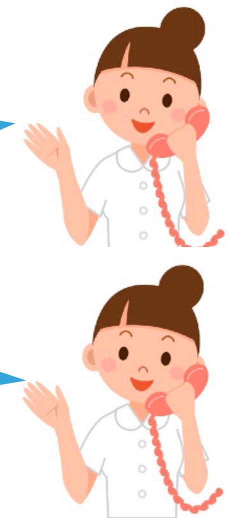
information structure

$P(s_1, s_2 \mid \theta)$

$s_1 \in S_1$

$s_2 \in S_2$

2 receivers



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maximize $E\left[u(p_1', p_2')\right]$ over feasible distributions of (p_1', p_2')

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$p'_2 = P(\theta = h \mid s_2)$

sender with utility

$u(p'_1, p'_2)$

THE PROBLEM

maximize $E\left[u(p'_1, p'_2)\right]$ over feasible distributions of (p'_1, p'_2)

WHAT IS KNOWN?

- $N = 1$ **is easy:** sender's value $= \text{cav}[u](p)$
  - Kamenica, Gentzkow (2011)

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WHAT IS KNOWN?

- $N = 1$ **is easy:** sender's value $= \mathrm{cav}[u](p)$
  - Kamenica, Gentzkow (2011)
- $N \geq 2$ **is hard:** feasible distributions can be complex
  - Arieli, Babichenko, Sandomirskiy, Tamuz (2021), Brooks, Frankel, Kamenica (2022)

OUR CONTRIBUTION

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CONDITIONING ON THE STATE SIMPLIFIES THE PROBLEM

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DEFINITION

μ^ℓ and μ^h on $[0,1]^2$ is a feasible pair of conditional distributions

$\iff$ $\exists$ information structure s.t. $(p_1', p_2') \sim \mu^\theta$ conditional on θ

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(μ^ℓ, μ^h) and (ν^ℓ, ν^h) with the same 1-dimensional marginals are feasible simultaneously

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MULTI-AGENT PERSUASION = OPTIMAL TRANSPORTATION PROBLEM!

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- **Archetypal coupling problem, many econ applications:**
  - Daskalakis et al. (2017), Kleiner, Manelli (2019), Boerma et al. (2021), Chiapporiet et al. (2010), Galichon (2021), Steinerberger, Tsyvinski (2019), Gensbittel (2015), Guo, Shmaya (2021), Cieslak, Malamud, Schrimpf (2021)

PERSUASION AS TRANSPORT

THEOREM

Value of a persuasion problem $\left(p, u^\ell, u^h\right)$ equals

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  - one-state, supermodular, submodular

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- **connection to extensive math transportation literature**
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  - one-state, supermodular, submodular
- **tractable dual extending 1-receiver results:**
  - $\mathrm{cav}[u]$-theorem by Kamenica, Gentzkow (2011) and duality by Dworczak, Kolotilin (2017)

THE DUAL

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Value of a persuasion problem (p, u^{ℓ}, u^{h}) equals

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admissible numbers V^{ℓ}, V^{h}

V^{l}, V^{h} are admissible $\iff \exists$ functions α_1, α_2 on $[0,1]$ s.t.

$$u^{\ell}(x,y) \le V^{l} + x \cdot \alpha_1(x) + y \cdot \alpha_2(y)$$

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  - gives a class of problems where full-information/partial-information signals are optimal

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THANK YOU!