On the Power of Mediation in Games

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The question

- NE: independent, decentralized play
- CE (Aumann, 1974): a mediator sends players possibly correlated recommendations
- multiple NE already make mediation useful for coordination
- when is correlation only equilibrium selection, and when does it do more?

When is $\operatorname{conv}(\mathrm{NE}) = \mathrm{CE}$ and when are they different?

A menu of motivations

The gap $\mathrm{CE}(G) \setminus \mathrm{conv}(\mathrm{NE}(G))$ is where correlation has genuine bite:

- **Design / recommendations:** can a recommender sustain outcomes beyond decentralized play?
- **Communication:** is pre-play talk just coordination on which NE to play? (Bárány, 1992; Forges, 1990; Lehrer and Sorin, 1997; Ben-Porath, 1998; Gerardi, 2004)
- **Learning:** can no-regret / calibrated dynamics find decentralized prediction? (Foster and Vohra, 1997; Fudenberg and Levine, 1999; Hart and Mas-Colell, 2000)
- **Computation:** when does a tractable LP describe a decentralized prediction? (Papadimitriou and Roughgarden, 2008)
- **Decision theory:** when does Bayesian rationality collapse to Nash logic? (Aumann, 1987)

What is known

CE **can be strictly larger than** $\mathrm{conv}(\mathrm{NE})$**:**

- the original 2×2 example (Aumann, 1974); all non-degenerate 2×2 games with two pure NE (Calvó-Armengol, 2006); mediation value in 2×2 games (Ashlagi, Monderer, and Tennenholtz, 2008)

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Extremality of a NE inside CE:

- always extreme in two-player games (Cripps, 1995; Evangelista and Raghavan, 1996; Canovas, Hansen, and Jaumard, 1999); non-extreme iff 3+ players mix (Rudov, Sandomirskiy, and Yariv, 2025)

Outline

1. Model: games, CE, NE, regularity
2. Characterization of games with $\mathrm{CE}(G) \setminus \mathrm{conv}(\mathrm{NE}(G)) \neq \emptyset$
3. How to construct points outside the hull
   - Tools: extremality and index theory

Model

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Correlated equilibrium (Aumann, 1974)

$\mu \in \Delta(A)$ is a CE if every recommendation is self-enforcing: for all i and all $a_i, a_i' \in A_i$

$$\sum_{a_{-i}} \mu(a_i, a_{-i}) \big[u_i(a_i, a_{-i}) - u_i(a_i', a_{-i}) \big] \ \geq \ 0$$

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- $\mathrm{CE}(G)$ is a polytope: finitely many linear inequalities on $\Delta(A)$
- an NE is a CE that is a product: $\mu = \mu_1 \times \dots \times \mu_n$
- $\mathrm{conv}(\mathrm{NE})$ = what a mediator reaches by only coordinating which NE to play

We need a non-degeneracy assumption

- pathological games blur the question
- if $u_i \equiv 0$ for all i, then $\mathrm{CE} = \mathrm{conv}(\mathrm{NE}) = \Delta(A)$
- correlation adds nothing for the wrong reason: there are too many NE

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- non-degenerate: the Jacobian of the indifference conditions on the support is invertible

- each NE is then stable: it persists, and moves smoothly, under small payoff perturbations
- Harsanyi (1973): all games except a measure-zero set are regular

Characterization

Main result: a sharp dichotomy

Theorem

Let G be regular. Then exactly one of the following holds:

$$\underbrace{\mathrm{CE} \setminus \mathrm{conv}(\mathrm{NE}) \neq \emptyset}_{\text{mediation has real bite}} \qquad \text{OR} \qquad \underbrace{\mathrm{CE} = \{\text{point}\}}_{\text{mediation powerless}}$$

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Interpretation

- communication and recommendation systems expand the set of decentralized outcomes whenever coordination failure is possible
- learning dynamics and LP solving are guaranteed to reach a decentralized outcome only in maximally robust games

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- CE (mediator knows nothing) $\subseteq$ communication equilibria (players report types) $\subseteq$ BCE (mediator omnipotent)
- $\Rightarrow$ the same dichotomy holds for communication equilibria and BCE

How to construct points outside the hull

Where do points outside the hull come from?

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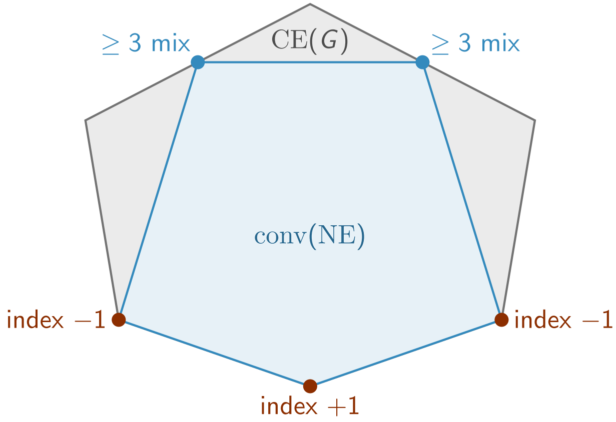
Two regimes require different arguments:

some NE has ≥ 3 mixers:	extremality-based argument
every NE has ≤ 2 mixers:	index-theory-based argument

The two ways to leave the hull (figure in dimension ≥ 7)



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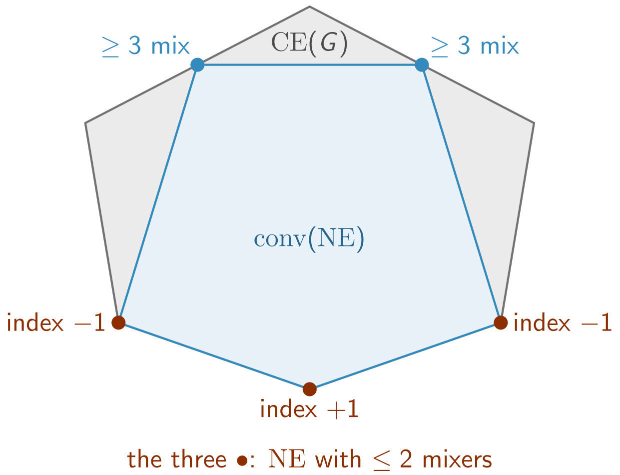

- NE with ≥ 3 mixers sit inside faces: corners of CE escape the hull
- NE with ≤ 2 mixers sit at vertices of CE (extreme)
    - index -1 $\Rightarrow$ admit a perturbation exiting the hull

Equilibria with three or more mixing players

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  - Formulated under regularity in the paper, but quasi-strictness is enough
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Details

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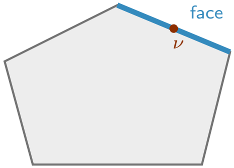
Let ν be a quasi-strict NE with ≥ 3 mixing players. Then no extreme point of the CE-face carrying ν is a quasi-strict NE



$\Rightarrow$ those extreme points lie outside $\mathrm{conv}(\mathrm{NE})$

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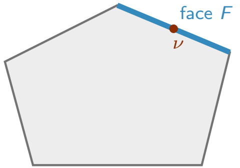

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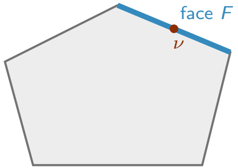

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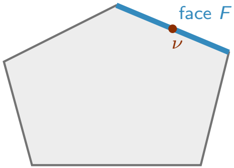

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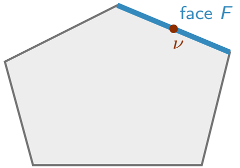

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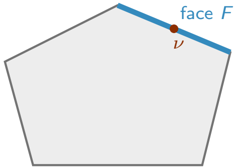

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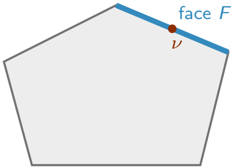

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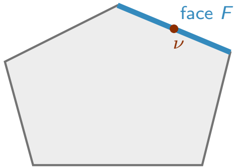

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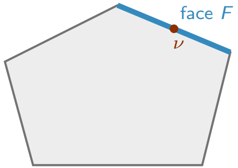

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- so e also has ≥ 3 mixers, contradicting extremality $\square$

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Two ideas to the rescue

1. anti-Nash: equilibria of the auxiliary game with $u_i' = -u_i$ can help construct incentive-compatible perturbations of an NE in the original game (Cripps, 1995)
2. index theory: the construction goes through for equilibria of index -1

Crash course in index theory

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- $J(\nu)$ is the Jacobian of all these, a $d \times d$ matrix, $d = \sum_i (m_i - 1)$, with zero diagonal blocks

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Properties

- regular $\Rightarrow \det J \neq 0 \Rightarrow \mathrm{index} = \pm 1$; pure $\mathrm{NE} \Rightarrow \mathrm{index} = +1$ (0×0 determinant)

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- regular $\Rightarrow \det J \neq 0 \Rightarrow \mathrm{index} = \pm 1$; pure $\mathrm{NE} \Rightarrow \mathrm{index} = +1$ (0×0 determinant)
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Crash course in index theory

For a regular $\mathrm{NE}\ \nu$,

$$\mathrm{index}_u(\nu) = \operatorname{sgn}\det J(\nu), \qquad \text{where } J \text{ is the Jacobian of the indifference equations}$$

- player i randomizes over m_i actions $\Rightarrow$ the $m_i - 1$ indifference equations:

$$u_i(a_i, \nu_{-i}) - u_i(b_i, \nu_{-i}) = 0, \qquad a_i \in \operatorname{supp}(\nu_i) \setminus \{b_i\} \quad \text{for some fixed } b_i$$

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- dropping them and reversing the sign of u: $\mathrm{index}_{-u}(\nu) = (-1)^d\, \mathrm{index}_u(\nu)$
- $\Rightarrow$ if only two players randomize, $d = 2(m-1)$ is even, so $\mathrm{index}_{-u}(\nu) = \mathrm{index}_u(\nu)$

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Index theorem (Gül, Pearce, and Stacchetti, 1993; Ritzberger, 1994; Govindan and Wilson, 1997; DeMichelis and Germano, 2000)

For a regular game, $$\sum_{\nu \in \mathrm{NE}} \mathrm{index}(\nu) = +1$$

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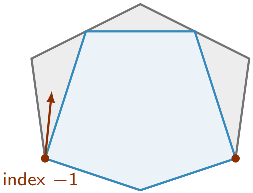
- if ν has index -1, then ν is mixed and there is another equilibrium $\nu' \neq \nu$

Constructing a CE outside the hull



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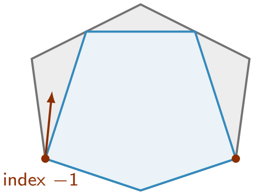

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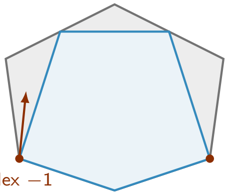

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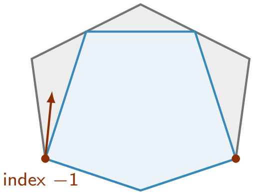

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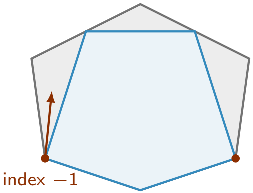

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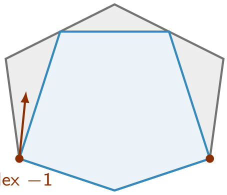

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Define,

$$\mu = \frac{\nu - \varepsilon\, \nu'}{1 - \varepsilon}$$

with $\varepsilon > 0$ small enough so that $\mu \geq 0$ and all outside-support constraints remain slack.
Then $\mu \in \mathrm{CE} \setminus \mathrm{conv(NE)}$

▸ Why outside

When do the two approaches overlap?

Remark

A version of the index-based construction works at any equilibrium ν with ≥ 3 mixers, as long as

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A puzzle: is the parity-index interplay the essence of the problem, or an artifact of the construction?

Conclusion

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Economic takeaway

- mediation, communication, and learning bring new outcomes relative to decentralized play whenever it leaves anything to coordinate on

Nerdy takeaway

- a nonlinear object, the NE set and its index, governs the local structure of a linear object, the CE polytope
- more surprising links likely to be found

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Thank you!

Why ≥ 3 mixers fails extremality?

Support bound: i is subject to $|A_i|(|A_i|-1)$ constraints. By Winkler (1988), an extreme CE has

$$\#\,\mathrm{supp}(\mu) \leq 1 + \sum_i |A_i|(|A_i|-1)$$

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Binary actions: $\leq 2n$ constraints $\Rightarrow$ extreme CE support $\leq 2n+1$. A NE with k mixers has support 2^k; dropping non-mixers,

$$2^k \leq 2k+1 \quad \Longrightarrow \quad k \leq 2$$

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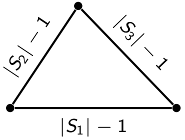
General supports: the polygon inequality. For any NE that is an extreme point of its Nash component, support sizes satisfy

$$|S_i| - 1 \leq \sum_{j \neq i} (|S_j| - 1) \quad \forall i$$

- each extra own action is one indifference constraint on opponents $\Rightarrow$ total constraints $\leq$ opponents' degrees of freedom
- Incompatible with the support bound once ≥ 3 mix ▸ Back

If NE is not extreme in its Nash component $\Rightarrow$ not extreme in CE



triangle inequality ($n=3$)

Appendix: why the anti-Nash perturbation escapes the Nash hull

Consider $\mu = \dfrac{\nu - \varepsilon\,\nu'}{1-\varepsilon}$, where ν' the anti-NE with $\mathrm{supp}(\nu')$ strictly inside $\mathrm{supp}(\nu)$

- $\Rightarrow \nu'$ omits some action $c_i \in \mathrm{supp}(\nu_i)$ for a player $i \in \{1,2\}$
- hence the player-i marginal increases at c_i:

$$\mu_i(c_i) = \frac{\nu_i(c_i)}{1-\varepsilon} > \nu_i(c_i)$$

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- suppose for contradiction $\mu = \sum_\ell \lambda_\ell\, \nu^\ell$ with $\nu^\ell \in \mathrm{NE}$, $\lambda_\ell > 0$ and $\sum \lambda_\ell = 1 \Rightarrow$ supp$(\nu^\ell) \subseteq$ supp(ν)
- at μ, conditional on c_i, player i is indifferent across supp(ν_i)
- $\Rightarrow$ the same holds under each ν^ℓ such that $\nu_i^\ell(c_i) > 0$; consider such ν^ℓ
- $\Rightarrow$ supp$(\nu_i^\ell) =$ supp(ν_i)
- $\Rightarrow$ supp$(\nu^\ell) =$ supp(ν) since $|\nu_{-i}^\ell| = |\nu_i^\ell| = |\nu_i| = |\nu_{-i}|$ by regularity
- in two-player games, no regular NE can share support with another NE, so $\nu^\ell = \nu$
- then $\mu_i(c_i) = \sum_\ell \lambda_\ell \nu_i^\ell(c_i) = \sum_\ell \lambda_\ell \nu_i(c_i) = (\sum_\ell \lambda_\ell)\,\nu_i(c_i) \leq \nu_i(c_i)$, contradiction

no such decomposition exists $\Rightarrow$ $\mu \in \mathrm{CE} \setminus \mathrm{conv}(\mathrm{NE})$

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